The random search problem: trends and perspectives

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PREFACE

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'I can't find my keys!' Who hasn't gone through this experience when leaving, in a hurry, to attend to some urgent matter? The keys could be in many different places. Unless one remembers where he or she has left the keys, the only solution is to look around, more or less randomly. Random searches are common because in many cases the locations of the specific targets are not known *a priori*. Indeed, such problems have been discussed in diverse contexts, attracting the interest of scientists from many fields, for example: the dynamical or stochastic search for a stable minimum in a complex energy landscape, relevant to systems such as glasses, protein (folding), and others; oil recovery from mature reservoirs; proteins searching for their specific target sites on DNA; animal foraging; survival at the edge of extinction due to low availability of energetic resources; automated searches of registers in high-capacity databases, search engine (e.g., 'crawlers') that explore the internet; and even pizza delivery in a jammed traffic system of a medium-size town. In this way, the subject is interesting, challenging and has recently become an important scientific area of investigation.

Although the applications are diverse, the underlying physical mechanisms are the same which will become clear in this special issue. Moreover, the inherent complexity of the problem, the abundance of ideas and methods found in this growing interdisciplinary field of research is studied in many areas of physics. In particular, the concepts and methods of statistical mechanics are particularly useful to the study of random searches. On one hand, it centres on how to find the global or local maxima of search efficiency functions with incomplete information. This is, of course, related to the long tradition in physics of using different conceptual and mathematical tools, such as variational methods, to extremize relevant quantities, e.g., energy, entropy and action. Such ideas and approaches are very important to solve computationally complex problems (e.g., protein folding), which involve optimizations in very high dimensional energy landscapes. On the other hand, random searches can also be studied from the perspective of diffusion and transport properties which is an important topic in condensed matter and statistical physics. For instance, the features of light scattered in a media, where the scatterers have a power-law distribution of sizes in many aspects, may resemble the patterns generated by a searcher performing Lévy walks.

There are many questions related to random searches: how the searcher moves or should move, what are the patterns generated during the locomotion, how do the encounter rates depend on parameters of the search, etc. But perhaps, the most well known issue is how to optimize the search for specific target scenarios. The optimization can be in either continuous or discrete environments, when the information available is limited. The answer to this question determines specific strategies of movement that would maximize some properly defined search efficiency measure. The relevance of the question stems from the fact that the strategy-dynamics represents one of the most important factors that modulate the rate of encounters (e.g., the encounter rate between predator and prey). In the general context, strategy choices can be essential in determining the outcome and thus the success of a given search. For instance, realistic searches — and locomotion in general — require the expenditure of energy. Thus, inefficient search could deplete energy reserves (e.g., fat) and lead to rates of encounters below a minimum acceptable threshold (resulting in extinction of a species, for example).

The framework of the random search 'game' distinguishes between the two interacting players in a context of pursuit and chance. They are either a 'searcher' (e.g., predator, protein, radar, 'crawler') or a 'target' (e.g., prey, DNA sequence, a missing aircraft, a given web site). Regarding the nature of the searching drive, in certain instances, it can be guided almost entirely by external cues, either by the cognitive (memory) or detective (olfaction, vision, etc) skills of the searcher. However, in many situations the movement is non-oriented, being in essence a stochastic process. Therefore, in such cases (and even when a small deterministic component in the locomotion exists) a random search effectively defines the final rates of encounters.

Hence, one reason underlying the richness of the random search problem relates just to the 'ignorance' of the locations of the randomly located targets. Contrary to conventional wisdom, the lack of complete information does not necessarily lead to greater complexity. As an illustrative example, let us consider the case of complete information. If the positions of all target sites are known in advance, then the question of what sequential order to visit the sites so to reduce the energy costs of locomotion itself becomes a rather challenging problem: the famous 'travelling salesman' optimization query, belonging to the NP-complete class of problems. The ignorance of the target site locations, however, considerably modifies the problem and renders it not amenable to be treated by purely deterministic computational methods. In fact, as expected, the random search problem is not particularly suited to search algorithms that do not use elements of randomness.

So, only a statistical approach to the search problem can adequately deal with the element of ignorance. In other words, the incomplete information renders the search under-determined, i.e., it is not possible to find the 'best' solution to the problem because all the information is not given. Instead, one must guess and probabilistic or stochastic strategies become unavoidable. Also, the random search problem bears a relation to reaction–diffusion processes, because the search involves a diffusive aspect, movement, as well as a reactive component, e.g., eating, mating, etc.

From the comments above, it is clear that the subject can be treated from the perspective of different fields and subfields of physics and mathematics: statistical mechanics, stochastic processes, Lévy walks and flights, complex systems, fractal geometry, and non-linear phenomena. Some important questions in random searches, especially in the case of discrete landscapes, are also associated with graph theory, random lattices, and complex networks.

The aim of this special issue is to bring together, in a single publication, all or most of the relevant theoretical concepts–ideas together with discussions of recent findings that are important for understanding the main elements of random searches. In addition, we will address the types of problems which are characteristic of random searching. Thus, we sincerely hope that this collection of works will provide a good overview for anyone interested in this field.

Finally, we should thank the editors and staff of *Journal of Physics A: Mathematical and Theoretical* for opining that random searches are an interesting topic of research deserving a topical publication. Furthermore, we are very grateful to Rebecca Gillan for helping us at all stages of preparation and organization of this special issue. Finally, we would like to thank the contributing authors who share with the guest editors the enthusiasm and interest for this fascinating field of research.

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